Exacntess and shifts in geometrical intelligibility: the case of the cycloid

Davide Crippa
Université Paris VII, Denis Diderot/REHSEIS

An account of the history of mathematics in early modern geometry cannot avoid considering fundamental patterns of change. The circulation of Pappus' edition among late XVIth century mathematicians, the spread of algebra as a representational apparatus apt for solving geometrical problems, and the successive developments of Newton's and Leibniz's differential and integral calculus are few but well known examples of transitions between practices occurred during XVIIth century.

In particular, a constant attitude underliying these interpractice transitions can be isolated: namely, the attempt at defining the contents and aims of «geometry », coinceived as a specific body of mathematical knowledge.

Following a suggestion from Henk Bos, the concept of "exactness" may be adopted as a norm in order to identify those procedures which, by a given mathematical agent within a given community, were accepted as leading to legitimate mathematical knowledge and to the geometrical intelligibility of its objects. Textual evidence documenting shifts in conceptions of geometrical exactness would allow to track down shifts in the content of "geometry".

In order to better understand this claim, I will start by considering the immediate heritage of the ideas contained in Descartes' *Geometria* (which was held as a reference for most, if not all XVIIth century scholars) with respect to a specific mathematical object: the cycloid. My aim will not be to inquire how cartesian techniques were applied (with different degree of proficiency) in order to study its properties, but rather what were the grounds to treat this curve as a mathematically intelligible object?

Beginning with Descartes' claim on its mechanical (and thus non exact and non geometrical) nature, I will oppose Pascal's and Leibniz's stances. Both mathematicians agreed that the cycloid was to be considered an exact curve, but they justified their positions recurring to opposite justificatory packages.

At the core of Pascal's argument, exposed in *Les Pensées* and in *Lettre de Dettonville à Carcavi*, lies the assumption that the method of indivisbles, propounded by Cavalieri and fruitfully applied by Pascal in studying the properties of the cycloid, is exact. Consequently, the curve itself could be described in a legitimate and intelligible manner, although at the prize of distorting the cartesian concept of exactness so as to subsume mechanics under it.

Leibniz studied the cycloid, among other places, in a tract from 1675 called *De Trochoeidis.*

There, he reached the same conclusions as Pascal, but he grouded them on an opposite justificatory package. Indeed, Leibniz did not see why the cycloid should not conform to the cartesian criterion of exactness, since, as every properly geometrical curve according to Descartes, it was generated "uno continuo moto". Thus, we can say that Leibniz accepted the cartesian criterion of exactness, but employed it with more generality.

Finally, these exemples contribute to show how the study of the same mathematical object or the adoption of the same mathematical methods can be grounded on different norms, which form an important part of a transition between practices.